

Interference Networks with Caches at Both Ends

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Abstract—A $K_T \times K_R$ cache-aided wireless interference network, in which both the transmitters and the receivers are equipped with cache memories is studied. Each user requests one file from a library of N popular files. The goal is to design the cache contents without the knowledge of the particular user demands, such that all possible demand combinations can be satisfied reliably over the interference channel. The achievable sum degrees-of-freedom (sDoF) and the normalized delivery time (NDT) are studied for centralized and decentralized network architectures, respectively. First, using a combination of interference alignment (IA), zero-forcing (ZF) and interference cancellation (IC) techniques, a novel caching and transmission scheme for centralized networks is introduced, and it is shown to improve the sDoF upon the state-of-the-art. Then, the NDT is studied when the content placement at the receiver caches is carried out in a decentralized manner. Our results indicate that, for this particular network architecture, caches located at the receiver side are more effective than those at the transmitter side in order to reduce the NDT.

Index terms— Cache-aided networks, interference management, degrees-of-freedom, zero-forcing, interference alignment.

I. INTRODUCTION

It is now widely accepted that the expected wireless traffic growth [1] can only be sustained by reducing the cell size and by enabling high-density spatial reuse of limited communication resources [2], [3]. A geographical area, which is covered by one macro-cell, will require thousands of femto-cells [4]. While this approach will reduce the congestion on the wireless access, exponentially growing traffic will increase the load on the back-haul links significantly. On the other hand, it is expected that by 2020 video content will constitute 75% of the overall mobile data traffic [1]. In order to exploit the features of video traffic and to overcome the bottleneck on the back-haul links, *content caching* has been proposed and studied extensively in the recent years [5]–[17]. The motivation for content caching is content reuse, i.e., many users request a small set of popular contents, and therefore, caching can replace, or reduce, the need for back-haul links. Cache-aided networks take advantage of distributed memories placed across the network (users devices, access points, routers,...) to bring content closer to end users. Networks have been traditionally passive in serving new user demands, that is, requests are processed only after they are received. However, in *content caching*, networks try to anticipate user requests in advance and place popular contents in the available cache memories, improving the quality-of-service for all the users.

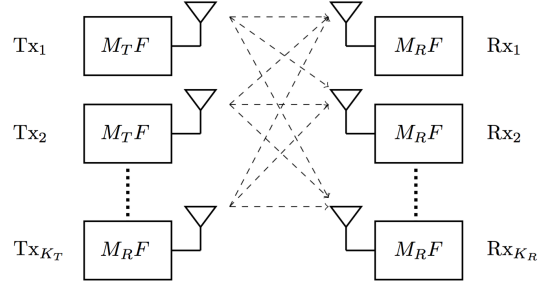


Fig. 1: The $K_T \times K_R$ wireless interference network with transmitter and receiver caches.

Many models and performance measures have been considered in the literature for cache-aided network design, all showing that caching can improve the network performance drastically. In their pioneering work [10], Maddah-Ali and Niesen consider a single server, connected through an error-free shared link to N users equipped with local caches. They propose a novel coded caching scheme that creates and exploits multicasting opportunities across the users, and show that it outperforms uncoded caching. In [9], [18] it is shown that the benefits of coded caching extends to the decentralized setting, in which users cache random bits.

The coded caching framework of [10] has recently been extended to an interference network by considering cache memories at the transmitter terminals [7]. Considering a 3×3 Gaussian interference network, the authors study the achievable sum degrees-of-freedom (sDoF). They propose a transmission scheme that exploits zero-forcing (ZF) and interference alignment (IA) techniques, leveraging content availability at the transmitter caches, in order to create different types of channels for delivering the remaining parts of the requested content. In [12], the authors extend this model to a $K_T \times K_R$ network, in which both the transmitters and receivers are equipped with cache memories. Nonetheless, in [12], the authors exploit only ZF and the receiver cache contents for interference cancellation (IC). The work in [13] studies the storage-latency trade-off for cache-aided wireless interference networks consisting of K_T transmitters ($K_T \geq 2$) and two or three receivers. Authors introduce a new coded caching technique that leverages adjustable file splitting, and using this placement phase as well as IA and ZF techniques, they characterize the fractional delivery time, obtaining similar

results to those in [7], [12]. A 3×3 cache-aided MIMO interference network is studied in [14], where the achievable fractional delivery time is characterized. Recently, the work in [19] provides a constant-factor approximation of the DoF that can be obtained in a $K_T \times K_R$ cache-aided wireless interference network with storage capabilities at both ends. The result is obtained leveraging a strategy that separates the physical and network layers. Authors show that, for this particular approach, ZF is not needed for order-optimal delivery. Finally, in [15], authors introduce the normalized delivery time (NDT) as a measure of performance for cache-aided networks. They characterize the NDT for 2×2 and 3×3 interference networks with caches only at the transmitter side. Lower-bounds on the NDT are also provided for these particular network configurations.

Similarly to [12] and [19], we consider a general interference network of single antenna terminals with cache capabilities at both ends. We first propose a centralized caching and delivery scheme that uses the same placement phase as in [12], but leverages a combination of ZF, IA and IC. We show that the proposed scheme achieves a higher sDoF than the one presented in [12], which does not exploit IA. Furthermore, in comparison with [13] and [14], where authors study the case of single antenna users in a $K_T \times 3$ network and multi-antenna terminals in a 3×3 network, respectively, our proposed scheme is a general solution to the cache-aided wireless interference networks for single antenna terminals, that is, we present a valid solution for any arbitrary number of transmitters, receivers and cache sizes. Besides, in contrast to [19] that presents an approximate characterization of the DoF, we provide a closed-form expression for the achievable sDoF.

Then, we look at the decentralized caching problem. This is a more realistic scenario as we consider the transmitters as static base stations whose cache contents can be centrally coordinated, whereas the receivers correspond to mobile users whose cache contents cannot be coordinated centrally, since we do not know in advance from which base station each user will request its content during the peak traffic period. We extend the proposed delivery scheme for the centralized scenario to the decentralized setting, and characterize the corresponding achievable NDT. Previous works [9], [18] have focused on coding algorithms for this type of networks in noiseless channels, but, to the best of our knowledge, this paper represents the first attempt to address the interference management problem in a decentralized cache-aided interference network.

The remainder of the paper is organised as follows. The system model is introduced in Section II. The achievable scheme for the centralized architecture is presented in Section III, while it is extended to a decentralized network in Section IV. Finally, our conclusions are summarized in Section V.

II. SYSTEM MODEL

We consider a wireless network with K_T transmitters $\{\text{Tx}_1, \text{Tx}_2, \dots, \text{Tx}_{K_T}\}$ and K_R receivers

$\{\text{Rx}_1, \text{Rx}_2, \dots, \text{Rx}_{K_R}\}$ (see Figure 1), and a library of N files $\{W_1, W_2, \dots, W_N\}$, each of which contains F bits. Each transmitter and each receiver is equipped with a cache memory of $M_T F$ and $M_R F$ bits, respectively. We impose the condition $K_T M_T + M_R \geq N$, so that the transmitters' caches are sufficient to collaboratively cache the files in the library that cannot be cached at each of the receivers.

The system operates in two phases; in the first phase, called the **placement phase**, the transmitter and receiver caches are filled without the knowledge of the future user demands. The cache contents of Tx_i at the end of the placement phase is denoted by \mathcal{P}_i , where $|\mathcal{P}_i| \leq M_T F$, $\forall i \in [K_T] \triangleq \{1, \dots, K_T\}$; while the cache contents of Rx_i is denoted by \mathcal{Q}_i , satisfying $|\mathcal{Q}_i| \leq M_R F$, $\forall i \in [K_R]$. In the **centralized scenario** the mapping function from the library to cache contents are known by all the nodes, while each transmitter knows only the contents of its own cache. On the other hand, in the **decentralized scenario**, coordination among users' cache contents is not possible; therefore, each user caches an equal number of bits randomly from each file in the library.

The receivers reveal their requests after the placement phase, and the **delivery phase** follows. Let W_{d_j} denote the file requested by Rx_j , $\forall j \in [K_R]$, and $\mathbf{d} \triangleq [d_1, \dots, d_{K_R}] \in [N]^{K_R}$ denote the vector of all user demands.

The delivery phase takes place over an independent and identically distributed (i.i.d.) additive white Gaussian noise interference channel. The signal received at Rx_j at time t is:

$$Y_j(t) = \sum_{i=1}^{K_T} h_{ji} X_i(t) + Z_j(t), \quad (1)$$

where $X_i(t) \in \mathbb{C}$ represents the signal transmitted by Tx_i at time t , $h_{ji} \in \mathbb{C}$ represents the channel coefficient between receiver j and transmitter i , and $Z_j(t)$ is the additive Gaussian noise term at Rx_j . We assume that the channel coefficients $\mathbf{H} \triangleq \{h_{i,j}\}_{i \in [K_R], j \in [K_T]}$, and the demand vector \mathbf{d} are known by all the transmitters and receivers.

Transmitter Tx_i , $\forall i \in [K_T]$, maps \mathbf{d}, \mathbf{H} , and its own cache contents \mathcal{P}_i to a channel input vector of length L , $\mathbf{X}_i = [X_i(1), \dots, X_i(L)]$. We impose an average power constraint P on each transmitted codeword, i.e., $\frac{1}{L} \|\mathbf{X}_i\|^2 \leq P$.

Receiver Rx_i , $\forall i \in [K_R]$, decodes its desired file W_{d_i} using \mathbf{d}, \mathbf{H} , its own cache content \mathcal{Q}_i , and the corresponding channel output $\mathbf{Y}_j = [Y_j(1), \dots, Y_j(L)]$. Let \hat{W}_i denote its estimate of W_{d_i} . The sum rate of this coding scheme is $K_T F/L$, while the error probability is defined as follows:

$$P_e = \max_{\mathbf{d} \in [N]^{K_R}} \max_{i \in [K_R]} \Pr(\hat{W}_i \neq W_{d_i}). \quad (2)$$

We say that rate $R(M_T, M_R, P)$ is achievable if there exists a sequence of coding schemes with sum rate $R(M_T, M_R, P)$ that achieves a vanishing probability of error, i.e., $P_e \rightarrow 0$ as $F, L \rightarrow \infty$. We let $C(M_T, M_R, P)$ denote the maximum achievable sum rate given the cache capacities and the power constraint. Then, the sumDoF (sDoF) of the system is defined as

$$\text{sDoF}(M_T, M_R) = \liminf_{P \rightarrow \infty} \frac{C(M_T, M_R, P)}{\log(P)}. \quad (3)$$

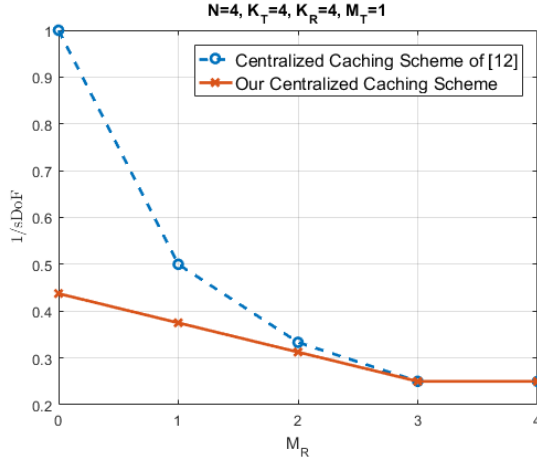


Fig. 2: Comparison of the inverse of the achievable sDoF in the decentralized scenario presented in this paper and the one in [12].

Note that, since each receiver has a single antenna, the DoF at each receiver is limited by one.

An alternative performance measure, called the normalized delivery time (NDT), is introduced in [15], in order to account for the latency in the system. The NDT is defined as the asymptotic delivery time per bit in the high-power, long-blocklength regime, that is,

$$\text{NDT} \triangleq \limsup_{F, P \rightarrow \infty} \frac{L}{F / \log P}. \quad (4)$$

We study the achievable sDoF in the centralized scenario, whereas in the decentralized scenario NDT is a more appropriate performance measure since the distribution of the different pieces of a file among receiver caches is asymmetric; which means that, achievable DoF is not uniform across a file, e.g., pieces of a file stored in more than one receiver can achieve a higher DoF than those stored at only one receiver.

III. CENTRALIZED CACHING

In this section, we develop a coding scheme that exploits ZF, IA and IC techniques jointly in a centralized cache-aided interference network with cache memories at both ends. The same scenario is also considered in [12], where the scope is limited to ZF and IC techniques.

A. Placement Phase

We use the same algorithm as in [12] for the placement phase, which divides each file into $\binom{K_T}{t_T} \binom{K_R}{t_R}$ equal-size non-overlapping subfiles, where $t_T \triangleq K_T M_T / N \in \mathbb{Z}$ and $t_R \triangleq K_T M_R / N \in \mathbb{Z}$. For file $A \in \{W_1, \dots, W_N\}$, the subfile stored at transmitters in X and receivers in Y is denoted by $A_{X,Y}$.

B. Delivery Algorithm

We explain the coding scheme for the delivery phase on a simple example. Consider the following network configuration: $K_T = 4$, $M_T = 2$, $K_R = 4$, $M_R = 1$, and $N = 4$ with the file library denoted by $\{A, B, C, D\}$. The bounds provided in [12] are not tight in this setting; the achievable sDoF is 3, while the upper-bound is 4. For this setting we have $t_T = \frac{K_T M_T}{N} = 2$ and $t_R = \frac{K_R M_R}{N} = 1$. File A is divided into $\binom{4}{2} \binom{4}{1} = 24$ subfiles as follows:

$$\begin{aligned} &A_{12,1}, A_{12,2}, A_{12,3}, A_{12,4}, A_{13,1}, A_{13,2}, A_{13,3}, A_{13,4}, \\ &A_{23,1}, A_{23,2}, A_{23,3}, A_{23,4}, A_{14,1}, A_{14,2}, A_{14,3}, A_{14,4}, \\ &A_{24,1}, A_{24,2}, A_{24,3}, A_{24,4}, A_{34,1}, A_{34,2}, A_{34,3}, A_{34,4}. \end{aligned}$$

The same division is applied also to files B , C and D . Note that each transmitter caches half of the subfiles, whereas each receiver caches a quarter of the subfiles, of each file; resulting in all caches to be fully utilized.

In the delivery phase, we assume, without loss of generality, that receivers 1, 2, 3 and 4 request files A , B , C and D , respectively (worst case demand). Each receiver already has 6 subfiles of its desired file in its cache, e.g., Rx_1 has cached subfiles $A_{12,1}$, $A_{13,1}$, $A_{23,1}$, $A_{14,1}$, $A_{24,1}$ and $A_{34,1}$. Transmitters should then provide the rest of the subfiles.

Let $A_{X,Y}^Z$ denote the subfiles of file A that have been cached by transmitters in X and receivers in Y , while Z denotes the receiver at which the transmitters in X zero-force their interference when transmitting this subfile. We first focus on the transmission of the following subfiles:

$$\begin{aligned} &A_{12,2}^3, A_{13,2}^3, A_{23,2}^3, A_{14,2}^3, A_{24,2}^3, A_{34,2}^3, \\ &B_{12,3}^4, B_{13,3}^4, B_{23,3}^4, B_{14,3}^4, B_{24,3}^4, B_{34,3}^4, \\ &C_{12,4}^1, C_{13,4}^1, C_{23,4}^1, C_{14,4}^1, C_{24,4}^1, C_{34,4}^1, \\ &D_{12,1}^2, D_{13,1}^2, D_{23,1}^2, D_{14,1}^3, D_{24,1}^2, D_{34,1}^2. \end{aligned} \quad (5)$$

Each of these subfiles is cached by a receiver that does not request it. Naturally, each of them is going to be zero-forced at a receiver that does not have it in the cache. For example, each subfile of file A in (5) is stored at Rx_2 and will be zero-forced at Rx_3 .

Consider the transmission of subfile $A_{12,2}^3$. This subfile is intended to be zero-forced at Rx_3 ; hence, Tx_1 and Tx_2 precode this file with the scaling factors h_{32} and $-h_{31}$, respectively. This subfile is going to be received at receivers Rx_1 , Rx_2 , Rx_3 and Rx_4 with the equivalent channel gains: $h_{11}h_{32} - h_{12}h_{31}$, $h_{21}h_{32} - h_{22}h_{31}$, $h_{31}h_{32} - h_{32}h_{31}$, $h_{41}h_{32} - h_{42}h_{31}$, respectively. These channel coefficients can be restated in terms of the channel matrix minors, thus, receivers Rx_1 , Rx_2 , Rx_3 and Rx_4 receive subfile $A_{12,2}^3$ with the scaling factors $M_{24,34}$, $M_{14,34}$, 0 and $M_{12,34}$, respectively¹.

We proceed by analyzing the interference caused by the transmission of subfile $A_{12,2}^3$. Rx_1 requests file A , thus subfile

¹ $M_{i,j}$ is the minor of the channel matrix \mathbf{H} . The minor of a square matrix A is the determinant of the submatrix formed by deleting the i -th row and j -th column of this matrix.

$A_{12,2}^3$ is intended for this receiver. Rx_2 can cancel the interference caused by this subfile, as it has already been cached by this receiver. Furthermore, this subfile is zero-forced at Rx_3 , meaning that no interference is caused at this particular receiver. This subfile, however, does cause interference at Rx_4 . We can assert that, based on the grouping of subfiles proposed above, transmission of the subfiles of file A in (5) will cause interference only at Rx_4 . Following a similar reasoning, the subfiles of file B , C and D , respectively, will cause interference only at Rx_1 , Rx_2 and Rx_3 . All these subfiles are transmitted simultaneously within the same channel block, and at this point, the potential benefits of IA in this setting are conspicuous. If we accomplish aligning all the interfering signals into the same subspace at each receiver, the DoF will be $6/7$ per receiver (see Figure 3).

Next, we present the construction of the IA scheme. As said before, the interference at Rx_1 is caused by subfiles $B_{12,3}^4$, $B_{13,3}^4$, $B_{23,3}^4$, $B_{14,3}^4$, $B_{24,3}^4$ and $B_{34,3}^4$, which arrive at the receiver with equivalent channel coefficients $M_{23,34}$, $M_{23,14}$, $M_{23,23}$, $M_{23,24}$, $M_{23,13}$ and $M_{23,12}$, respectively. The goal is to align all these interfering signals at the receiver. To do so, *real IA* techniques presented in [20] and [7] are leveraged, which show that transmitters are able to align the interfering subfiles at the receiver (due to space limitation, proofs are omitted and will be available in an extended version). Similarly, the interfering signals at Rx_2 , Rx_3 and Rx_4 can also be aligned. Note that 6 subfiles are transmitted and all the interfering signals collapse into one dimension (see Figure 3).

By carefully grouping the remaining subfiles, a similar transmission scheme can be exploited. We now proceed with the transmission of the subfiles $A_{X,3}^4$, $B_{X,4}^1$, $C_{X,1}^2$, $D_{X,2}^3$, where X represents the transmitters at which each subfile is cached. We can see that each receiver is interfered by only one of those groups of subfiles: Rx_2 will be interfered only by subfiles $A_{X,3}^4$; subfiles in $B_{X,4}^1$ will cause interference only at Rx_3 , and $C_{X,1}^2$ and $D_{X,2}^3$ will cause interference only at Rx_4 and Rx_1 , respectively. Now, by leveraging the same IA scheme explained above, we can guarantee that all the interference collide in the same sub-space.

Subfiles $A_{X,4}^2$, $B_{X,1}^3$, $C_{X,2}^4$ and $D_{X,3}^1$ can be transmitted over a third channel block in a similar fashion. Note that, within each channel block the transmission of the desired subfiles at each receiver takes $6/7$ of the achievable unit DoF. Hence, we can achieve a per user DoF of $6/7$, while the sDoF is $24/7 = 3.42$.

We highlight that the proposed mixed-approach of IA, ZF and IC implemented in this paper achieves the highest sDoF in the literature for the above 4×4 scenario. A comparison of the achievable $1/sDoF$ for the 4×4 network between our proposed scheme and the one in [12] for different receiver cache size M_R is presented in figure 2 (we use the reciprocal $1/sDoF$ because is a convex function of M_T and M_R). Furthermore, our scheme does not require the use of scaling factors as in [7]. In order to guarantee independence, different channel coefficients are exploited, thus, IA is provided by the

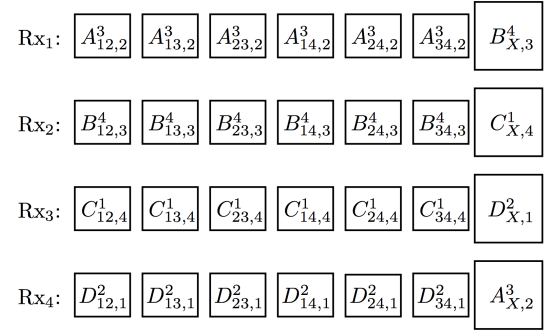


Fig. 3: Alignment of the subfiles in (5) at different receivers.

channel for free if the proper signal constellations are chosen. It must be said that, if we had used scaling factors, a better DoF can potentially be achieved. For example, in the 3×3 scenario with no caches at the receivers, it is shown in [7] that $sDoF = 18/7$ is achievable, while our scheme achieves a sDoF of $9/4$. However, our focus here is to provide a general scheme that exploits a mixed approach of IA, ZF and IC, and the generalization of the scaling factors introduced in [7] is cumbersome.

C. Main result

Here we present the sDoF result for a general scenario. We skip the proof due to space limitations, but it follows from similar arguments as the example given above.

Theorem 1. *For a centralized caching network with a library of N files, K_T transmitters, and K_R receivers, with cache capacities of M_T and M_R files, respectively, the following sDoF is achievable if $t_T, t_R \in \mathbb{Z}$:*

$$\min \left\{ \frac{\binom{K_T}{t_T} K_R}{\binom{K_T}{t_T} + K_R - t_T - t_R}, K_R \right\}. \quad (6)$$

The factor $\binom{K_T}{t_T}$ represents the number of orthogonal subspaces that the desired signals span at each receiver (K_R receivers in total). The denominator corresponds to the total number of orthogonal directions received (desired signals plus interference) $\binom{K_T}{t_T} + K_R - 1$, the ZF technique allows us to reduce the number of required orthogonal directions by $t_T - 1$. We can further reduce it by t_R thanks to the information stored in the receivers' caches.

Below, the implication of these results are discussed:

- Unlike the results presented in [12], the sDoF achieved by our scheme is not proportional to the aggregate cache size of the network. On the contrary, caches located at the transmitters are more valuable than those at the receivers.
- The fact that the caches at transmitter side lead to a higher sDoF signifies that employing IA and ZF jointly,

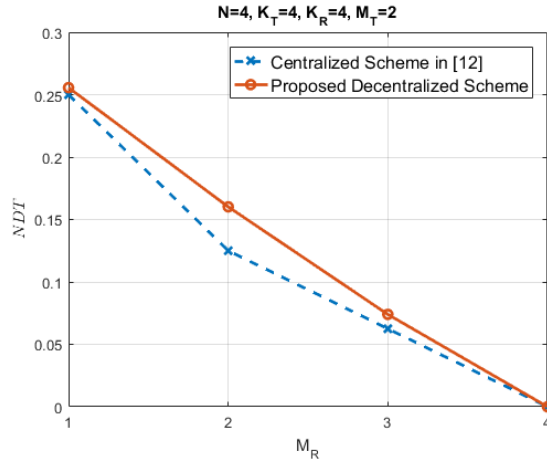


Fig. 4: Comparison of the achievable NDT between the decentralized scheme presented in this paper and the centralized scheme of [12].

by exploiting transmitters' cache, is more effective than IC using receivers' cache.

- Our results match the ones provided in [12] when their bound is tight ($\min\{t_T + t_R, K_R\} = K_R$).

IV. DECENTRALIZED CACHING

In this section we develop a coding scheme for interference management in decentralized cache-aided wireless networks.

A. Placement Phase

In the decentralized scenario, we need to leverage a placement technique that allows us to exploit broadcasting opportunities without relying on the number or the identity of the users that participate in the delivery phase. Therefore, we use a random placement technique (similarly to [9] and [11]), where each receiver fills its cache with randomly chosen $\frac{M_R}{N}F$ bits from each file. On the transmitter site, as before, each file is divided into $\binom{K_T}{t_T}$ disjoint partitions of equal size, each of which is stored in t_T transmitters. Again, we assume $t_T, t_R \in \mathbb{Z}$. Each of these partitions can be further divided into $\sum_{j=0}^{K_R} \binom{K_R}{j}$ subfiles, each corresponding to those bits that are stored by a particular subset of the receivers. Overall, we have a total of

$$\binom{K_T}{t_T} \sum_{j=0}^{K_R} \binom{K_R}{j} \quad (7)$$

disjoint subfiles.

Unlike in the centralized scenario, subfiles in (7) are not of equal size. By the law of large numbers, the size of the subfile that has been cached by t receivers can be approximated by

$$\left(\frac{M_R}{N}\right)^t \left(1 - \frac{M_R}{N}\right)^{K_R-t} F \text{ bits.} \quad (8)$$

B. Delivery Algorithm

We consider the following setup as an example: $K_R = 3$, $M_R = 1$, $K_T = 3$, $M_T = 2$ and $N = 3$, with the library of $\{A, B, C\}$. According to the decentralized placement phase explained before, file A is divided into 24 subfiles as follows:

$$\begin{aligned} &A_{12,0}, A_{12,1}, A_{12,2}, A_{12,3}, A_{12,12}, A_{12,13}, A_{12,23}, A_{12,123}, \\ &A_{13,0}, A_{13,1}, A_{13,2}, A_{13,3}, A_{13,12}, A_{13,13}, A_{13,23}, A_{13,123}, \\ &A_{23,0}, A_{23,1}, A_{23,2}, A_{23,3}, A_{23,12}, A_{23,13}, A_{23,23}, A_{23,123} \end{aligned}$$

The same subdivision is also applied to files B and C . Assume that files A , B and C are requested by R_{x1} , R_{x2} and R_{x3} , respectively. Notice that each receiver already has 12 subfiles of each file, e.g., R_{x1} has in its cache subfiles: $A_{12,1}$, $A_{12,12}$, $A_{12,13}$, $A_{12,123}$, $A_{13,1}$, $A_{13,12}$, $A_{13,13}$, $A_{13,123}$, $A_{23,1}$, $A_{23,12}$, $A_{23,13}$ and $A_{23,123}$.

First, we analyze the transmission of the subfiles that have been cached by only one receiver. For these subfiles ZF, IA and IC approaches can be exploited. Consider the subfiles $A_{12,2}$, $A_{23,2}$, $A_{13,2}$, $B_{12,3}$, $B_{23,3}$, $B_{13,3}$, $C_{12,1}$, $C_{23,1}$ and $C_{13,1}$. Each of the subfiles of file A is transmitted in a different channel block (i.e., $A_{12,2}$, $B_{23,3}$ and $C_{13,1}$ in the first, $A_{23,2}$, $B_{13,3}$ and $C_{12,1}$ in the second, and $A_{13,2}$, $B_{13,3}$ and $C_{23,1}$ in the third). Their interference is zero-forced at R_{x3} , by T_{x1} and T_{x2} . Their interference can be subtracted at R_{x2} , so they do not cause any interference at any receiver. Similarly, the interference from B 's subfiles is zero-forced at R_{x3} , and cancelled by R_{x1} . Finally, the interference from C 's subfiles is zero-forced at R_{x2} , and cancelled by R_{x1} . So, in this particular case, there is no need for IA as all the signals are received interference-free. Subfiles $A_{12,3}$, $A_{23,3}$, $A_{13,3}$, $B_{12,3}$, $B_{23,3}$, $B_{13,3}$, $C_{12,2}$, $C_{23,2}$ and $C_{13,2}$ are transmitted in a similar fashion. The sDoF for these subfiles is the maximum achievable, which is 3.

The transmission of subfiles $A_{12,23}$, $A_{23,23}$, $A_{13,23}$, $B_{12,13}$, $B_{23,13}$, $B_{13,13}$, $C_{12,12}$, $C_{23,12}$ and $C_{13,12}$, cached at two receivers, is even simpler, as their interference can be cancelled by two receivers, so, they can be simply broadcasted. Interference from A 's subfiles can be cancelled by both R_{x2} and R_{x3} , B 's subfiles by both R_{x1} and R_{x3} , and C 's subfiles by both R_{x1} and R_{x2} , so desired subfiles are received interference-free. The overall sDoF that can be obtained for this particular group of subfiles is 3, which, again, is the maximum.

At this point, the only remaining subfiles that must be transmitted are those that have not been cached by any receiver. For transmitting these subfiles IA and ZF approaches will be simultaneously exploited. R_{x1} is interested in three such subfiles, $A_{12,0}$, $A_{13,0}$ and $A_{23,0}$. At the same time, it receives interference from $B_{12,0}$, $B_{13,0}$, $B_{23,0}$, $C_{12,0}$, $C_{13,0}$ and $C_{23,0}$. However, part of these interfering signals can be eliminated using ZF as the subfiles are available at two transmitters. Following the notation of the previous section, we rewrite the signals that will be transmitted as: $A_{12,0}^2$, $A_{13,0}^2$, $A_{23,0}^2$, $B_{12,0}^3$, $B_{13,0}^3$, $B_{23,0}^3$, $C_{12,0}^1$, $C_{13,0}^1$ and $C_{23,0}^1$. Thus, the only interfering signals at R_{x1} are due to $B_{12,0}^3$, $B_{13,0}^3$ and $B_{23,0}^3$. We align these interfering signals by using the same IA approach as in

Section III. Therefore, at R_{x1} , subfiles of file A are received in different orthogonal dimensions, while the interference signals collapse to the same orthogonal dimension; that is, each subfile carries $\frac{1}{4}$ DoF, and the total DoF for R_{x1} is $\frac{3}{4}$. The same methodology can be applied to the remaining subfiles cached by only a single receiver. The sDoF for these subfiles is found to be DoF = $\frac{9}{4}$.

Then, the overall NDT can be evaluated as:

$$\text{NDT} = 3 \left(\frac{3 \cdot \left(\frac{2}{3}\right)^3}{\frac{9}{4}} + \frac{2 \cdot \frac{1}{3} \cdot \frac{4}{9} + \frac{1}{9} \cdot \frac{2}{3}}{3} \right) = \frac{147}{95}.$$

Figure 4 shows a comparison between the centralized delivery scheme presented in [12] and the proposed decentralized scheme, displaying that both schemes are very close in performance.

C. Main Results

The following theorem, stated here without proof, provides an achievable NDT performance for a general network.

Theorem 2. *For a decentralized network with K_T transmitters, K_R receivers, a library of N files, and cache size of M_T and M_R files at the transmitters and receivers, respectively, the following NDT is achievable for $t_T \in \mathbb{Z}$:*

$$K_R \sum_{t=0}^{K_R-1} \frac{\left(\binom{K_R}{t} - t \right) \left(\frac{M_R}{N} \right)^t \left(1 - \frac{M_R}{N} \right)^{K_R-t}}{\min \left\{ \frac{K_T K_R}{K_T + K_R - t_T - t}, K_R \right\}} \quad (9)$$

$$+ \frac{K_R M_T}{\min \left\{ K_R, \frac{K_T K_R}{K_T + K_R - t_T} \right\}} \cdot (\max \{M_R, 1\} - M_R).$$

In the first part of the expression, the numerator represents the number of subfiles that need to be transmitted multiplied by the expected length. The denominator embraces the sDoF that can be achieved when a combination of IA, ZF, and IC are exploited. It should be noticed that the number of dimensions spanned by the interfering signals vary depending on the subfiles that are transmitted. Finally, the second part of the expression corresponds to the case with $M_R = 0$. The parameter t represents the gain from cancelling the interference using the receivers' cache contents, while the factor t_T represents the interference cancellation due to ZF of those files that are available at more than one transmitter.

- The particular value of t_R , as long as it is greater than 0, does not affect the achievable sDoF, but it reduces the size of the packets that need to be transmitted.
- The benefit of receiver's caches is greater than the caches at the transmitter side as it reduces the size of the subfiles that need to be transmitted, and hence, the NDT.

The non corner points of the NDT curve, which represent the non integer values of t_T and t_R , can be achieved through memory-sharing [10].

V. CONCLUSIONS

In this work, novel caching and transmission schemes for interference management in centralized and decentralized wireless cache-aided interference networks are presented. The proposed schemes make use of ZF and IA techniques as well as the users' cache contents for IC. In the case of a centralized caching network, the proposed cache-aided transmission scheme achieves the highest sDoF in the literature. To the best of our knowledge, this is the first work to consider the NDT performance in a decentralized interference network, in which the cache contents of the receivers cannot be coordinated. By extending the proposed transmission strategy to the decentralized setting, we proposed an achievable NDT performance, and characterized its performance for a general network.

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